nag_kelvin_kei (s19adc)

1. Purpose

nag_kelvin_kei (s19adc) returns a value for the Kelvin function $\mathrm{kei}\, x$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_kelvin_kei(double x, NagError *fail)
```

3. Description

This function evaluates an approximation to the Kelvin function $\mathrm{kei}\, x$.

The function is based on several Chebyshev expansions.

For large x, $\mathrm{kei}\, x$ is so small that it cannot be computed without underflow and the function fails.

4. Parameters

x
Input: the argument x of the function.
Constraint: $\mathbf{x} \geq 0$.

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_GT
On entry, **x** must not be greater than ⟨*value*⟩: $\mathbf{x} = \langle value \rangle$.
x is too large, and the result underflows and the function returns zero.

NE_REAL_ARG_LT
On entry, **x** must not be less than 0.0: $\mathbf{x} = \langle value \rangle$.
The function is undefined and returns zero.

6. Further Comments

Underflow may occur for a few values of x close to the zeros of $\mathrm{kei}\, x$, which causes failure **NE_REAL_ARG_GT**.

6.1. Accuracy

Let E be the absolute error in the result, and δ be the relative error in the argument. If δ is somewhat larger than the machine representation error, then we have $E \simeq |x(-\mathrm{ker}_1 x + \mathrm{kei}_1 x)/\sqrt{2}|\, \delta$.

For small x, errors are attenuated by the function and hence are limited by the ***machine precision***.

For medium and large x, the error behaviour, like the function itself, is oscillatory and hence only absolute accuracy of the function can be maintained. For this range of x, the amplitude of the absolute error decays like $\sqrt{\pi x/2}e^{-x/\sqrt{2}}$, which implies a strong attenuation of error. Eventually, $\mathrm{kei}\, x$, which is asymptotically given by $\sqrt{\pi/2x}e^{-x/\sqrt{2}}$, becomes so small that it cannot be calculated without causing underflow and therefore the function returns zero. Note that for large x, the errors are dominated by those of the **math library** function exp.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 9.9 p 379.

7. See Also

nag_kelvin_ber (s19aac)
nag_kelvin_bei (s19abc)
nag_kelvin_ker (s19acc)

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_kelvin_kei(s19adc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s19adc Example Program Results\n");
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s19adc(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s19adc Example Program Data
             0.0
             0.1
             1.0
             2.5
             5.0
             10.0
             15.0
```

8.3. Program Results

```
s19adc Example Program Results
     x           y
   0.000e+00  -7.854e-01
   1.000e-01  -7.769e-01
   1.000e+00  -4.950e-01
   2.500e+00  -1.107e-01
   5.000e+00   1.119e-02
   1.000e+01  -3.075e-04
   1.500e+01   7.963e-06
```